“The number Pi, Isaac Newton and the graffiti of Woolsthorpe Manor”

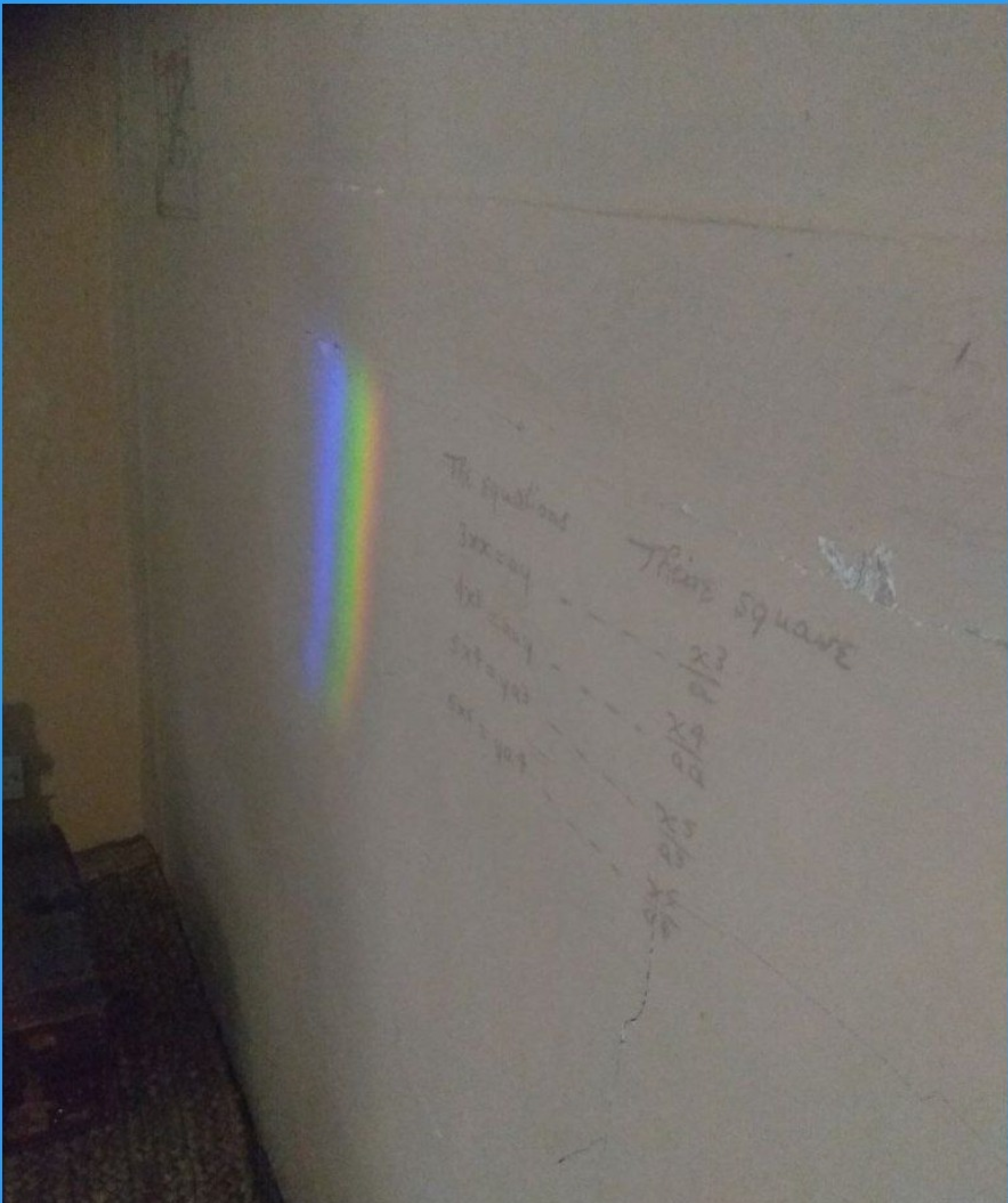
Time 15 Mar 2024 05:00 PM

Kyiv https://us05web.zoom.us/j/86075119007?pwd=O02tCkOe7WMjbID1bc2SkUO22uak34.1

The International Day of Mathematics (IDM) is a fantastic opportunity to celebrate the beauty of mathematics. Join us for an open online meeting of the mathematical circle and immerse yourself in the wonders of this captivating field! It’s a day to connect with fellow math enthusiasts from around the world, share knowledge, and explore the endless possibilities that mathematics offers. Whether you’re a student, educator, or simply someone who appreciates the power of numbers, this event promises to be an inspiring and enlightening experience. Let’s come together and embrace the magic of mathematics!

«Мені соромно сказати вам, до скількох цифр я вів ці розрахунки, не маючи на той час інших справ».Ісаак Ньютон, особистий щоденник, 1666 рік

INTERNATIONAL DAY OF
MATHEMATICS
MARCH 14


The equations
Their square

The equations

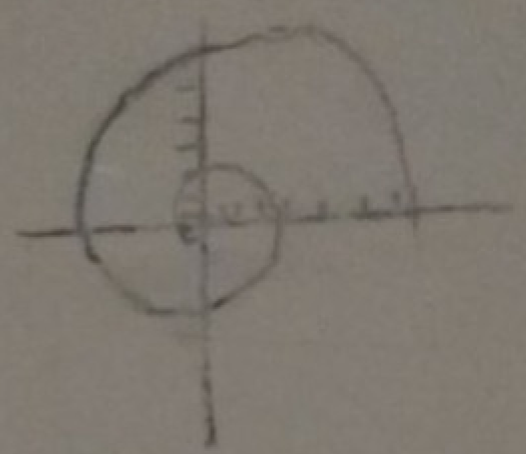
3xx = ay

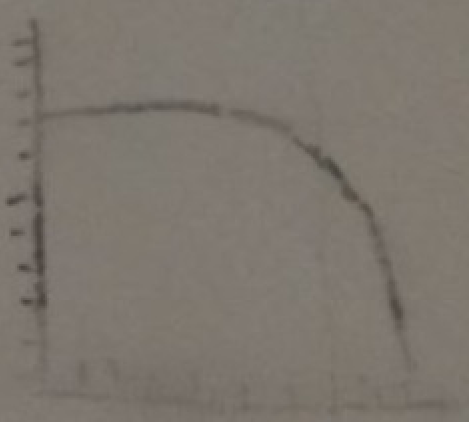
4x3 = aay

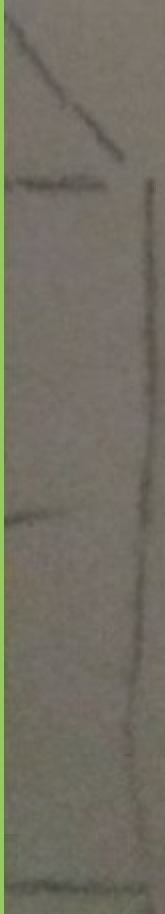
5x4 = ya3

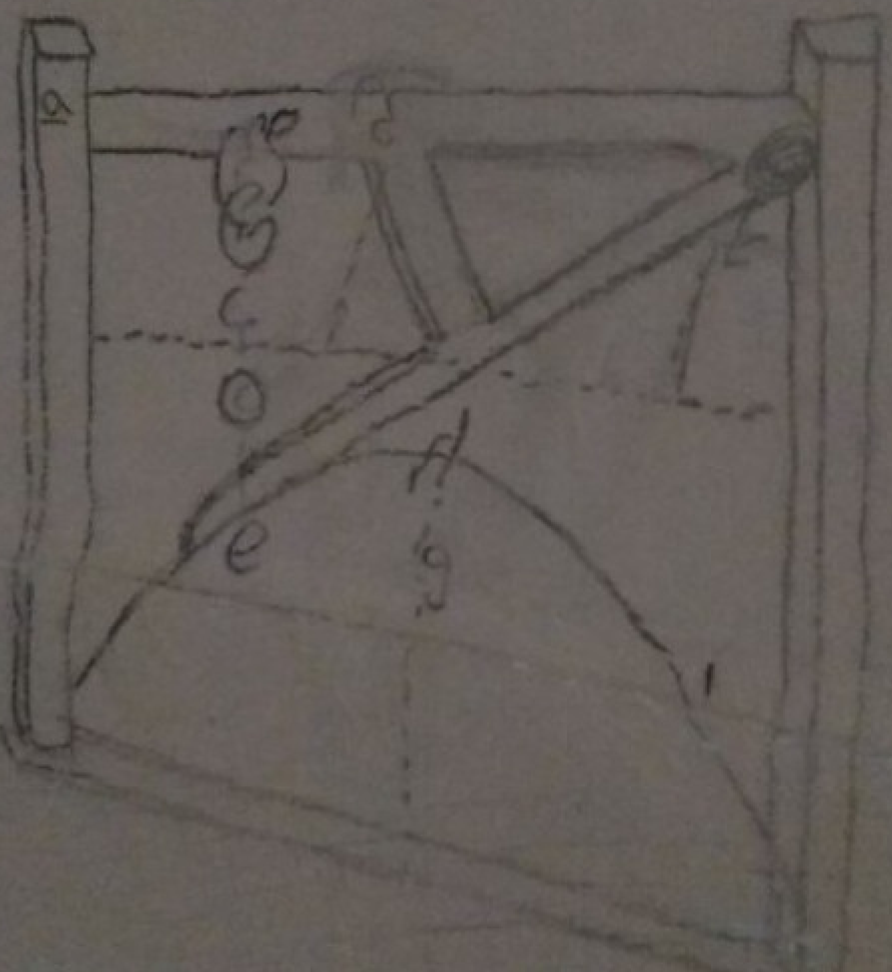
6x5 = ya4

The equations	Their squa
$3xx = ay$	$\frac{x^3}{a}$
$4x^3 = aay$	$\frac{x^4}{aa}$
$5x^4 = ya^3$	$\frac{x^5}{a^3}$
$6x^5 = ya^4$	$\frac{x^6}{a^5}$









a
c
d
e
g

TEACHER NOTES AND SOLUTIONS

In 1666 Newton found Pi to sixteen decimal places by evaluating the first twenty-two terms of this sum:

$$\pi = \frac{3\sqrt{3}}{4} + 24\,(\frac{1}{12} - \frac{1}{5 \times 2^5} - \frac{1}{28 \times 2^7} - \frac{1}{72 \times 2^9} \dots)$$

For Pi Day 2020 Matt Parker of Think Maths followed in Newton's footsteps and evaluated terms of this sum by hand to get an approximation for Pi. Matt had help from schools across the world who evaluated some of the terms for him – a total of twenty terms were evaluated, almost matching Newton's twenty-two. It truly was a mass participation calculation!

Deriving Newton's Sum

At Think Maths we investigated where this sum came from and discovered that Newton's derivation of the expression is surprisingly satisfying and accessible for **A-level students**.

The derivation involves the topics of: equation of a circle, binomial expansion, finding definite integrals, areas of sectors of circles, index laws, trigonometry, Pythagoras' theorem.

Challenge your students to derive Newton's sum for Pi. We've made a **sheet 'Newton's Approximation for Pi'** that you can use to **introduce the problem to students**. We haven't provided much structure – this is so teachers can decide how much structure/hints it is best to give their own students. **See our solutions/the full derivation on the page below.**

Evaluating Newton's Sum

Alternatively, perhaps your whole school/multiple classes could work together to evaluate by hand as many terms of the sum as they can.

Newton evaluated the first twenty-two terms of the sum, and Matt Parker evaluated the first twenty terms. Can your school match Matt's work? We've written out the first twenty terms of the sum on the page below for your use.

Students could watch Matt's video first to get ideas for how to do the written calculations.

38. To prove this theorem, let us write

$$\tan\frac{\varphi}{\omega} = \frac{M}{P},$$

such that M and P are quantities expressed in an arbitrary way, even, if you like, by decimal sequences, which always can happen, even when M, P are integers, because we have only to multiply each of them by an irrational quantity. We can also, if we like, write

$$M = \sin\frac{\varphi}{\omega}, \quad P = \cos\frac{\varphi}{\omega},$$

as above. And it is clear that, even if $\tan \varphi/\omega$ were rational, this would not necessarily hold for $\sin \varphi/\omega$ and $\cos \varphi/\omega$.

39. Since the fraction M/P exactly expresses the tangent of φ/ω, it must give all the quotients ω, 3ω, 5ω, etc., which in the present case are

$$+\frac{\omega}{\varphi}, \quad -\frac{3\omega}{\varphi}, \quad +\frac{5\omega}{\varphi}, \quad -\frac{7\omega}{\varphi}, \quad \text{etc.}$$

40. Hence, if the tangent of φ/ω is rational, then clearly M will be to P as an integer μ is to an integer π, such that, if μ, π are relatively prime, we shall have

$$M : \mu = P : \pi = D,$$

and D will be the greatest common divisor of M, P. And since reciprocally

$$M : D = \mu, \quad P : D = \pi,$$

we see that, since M, P are supposed to be irrational quantities, their greatest common divisor will be equally an irrational quantity, which is the smaller, the larger the quotients μ, π are.

41. Here are therefore *the two suppositions of which we must show the incompatibility*. Let us first divide P by M, and the quotient must be $\omega : \varphi$. But since $\omega : \varphi$ is a fraction, let us divide φP by M, and the quotient ω will be the φ-tuple of $\omega : \varphi$. It is clear that we could divide it by φ if we wished to do so. This is not necessary, since it will be sufficient that ω be an integer. Having thus obtained ω by dividing φP by M, let the residue be R'. This residue will equally be the φ-tuple of what it would have been, and that we have to keep in mind. Now, since $P : D = \pi$, an integer, we still have $\varphi P : D = \varphi\pi$, an integer. Finally, $R' : D$ will also be an integer. Indeed, since

$$\varphi P = \omega M + R',$$

we shall have

$$\frac{\varphi P}{D} = \frac{\omega M}{D} + \frac{R'}{D}.$$

Modular equations and approximations to π

Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

1. If we suppose that

$$(1+e^{-\pi\sqrt{n}})(1+e^{-3\pi\sqrt{n}})(1+e^{-5\pi\sqrt{n}})\cdots = 2^{\frac{1}{4}}e^{-\pi\sqrt{n}/24}G_n \tag{1}$$

and

$$(1-e^{-\pi\sqrt{n}})(1-e^{-3\pi\sqrt{n}})(1-e^{-5\pi\sqrt{n}})\cdots = 2^{\frac{1}{4}}e^{-\pi\sqrt{n}/24}g_n, \tag{2}$$

then G_n and g_n can always be expressed as roots of algebraical equations when n is any rational number. For we know that

$$(1+q)(1+q^3)(1+q^5)\ldots = 2^{\frac{1}{6}}q^{\frac{1}{24}}(kk')^{-\frac{1}{12}} \tag{3}$$

and

$$(1-q)(1-q^3)(1-q^5)\cdots = 2^{\frac{1}{6}}q^{\frac{1}{24}}k^{-\frac{1}{12}}k'^{\frac{1}{6}}. \tag{4}$$

Now the relation between the moduli k and l, which makes

$$n\frac{K'}{K} = \frac{L'}{L},$$

where $n = r/s, r$ and s being positive integers, is expressed by the modular equation of the rsth degree. If we suppose that $k = l', k' = l$, so that $K = L', K' = L$, then

$$q = e^{-\pi L'/L} = e^{-\pi\sqrt{n}},$$

and the corresponding value of k may be found by the solution of an algebraical equation. From (1), (2), (3) and (4) it may easily be deduced that

$$g_{4n} = 2^{\frac{1}{4}}g_nG_n, \tag{5}$$

$$G_n = G_{1/n},\ 1/g_n = g_{4/n}, \tag{6}$$

$$(g_nG_n)^8(G_n^8 - g_n^8) = \frac{1}{4}. \tag{7}$$

I shall consider only integral values of n. It follows from (7) that we need consider only one of G_n or g_n for any given value of n; and from (5) that we may suppose n not divisible by 4. It is most convenient to consider g_n when n is even, and G_n when n is odd.

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ORIGINAL PAPER

A Brief History of the Most Remarkable Numbers π, g and δ in Mathematical Sciences with Applications

Lokenath Debnath[1]

Published online: 1 April 2015

Abstract This paper deals with a brief history of the most remarkable numbers π, g and δ in mathematical sciences with their many examples of applications. Several series, products, continued fractions and integral representations of π are discussed with examples. The celebrated Newton method of approximation of π to many decimal places is included. The appearance of π in many problems, formulas, elliptic integrals and in probability and statistics is presented with examples of applications including the Tchebycheff problem of prime numbers, the Buffon needle problem and the Euler quadratic polynomial. The golden number g and its applications to algebra and geometry are briefly discussed. The Feigenbaum universal constant, δ is discovered in 1978 and it is found to occur in many period doubling bifurcation phenomena in the celebrated logistic map and the Lorenz differential equation system with chaotic (or aperiodic) solutions. Included is a numerically computed Lorenz attractor which resembles a *butterfly* or *figure eight*. The Lorenz attractor is a *strange attractor* because it has a non-integer (or fractal) dimension. The major focus of this article is to provide basic pedagogical information through historical approach to mathematics teaching and learning of the fundamental knowledge and skills required for students and teachers at all levels so that they can understand the concepts of mathematics, and mathematics education in science and technology and pursue further research.

Keywords Universal constant π · Golden number g · Feigenbaum's constant · δ · Chaos

Mathematics Subject Classification 01A · 47A07 · 26D15

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty–a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only greatest art can show."
Bertrand Russell

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